

# Optimum Life Test Plan for Accelerated Deterioration Test of Three Stress Levels

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**Abstract**  
Accelerated degradation tests are often used to quickly find the life of materials such as electrical insulators. An ADT consists of several high levels of stress and the number of samples at that level. Each sample is subjected to the stress for a certain evaluation time and the degradation rate at that time is measured. The time when the deterioration reaches a certain threshold is regarded as the failure time. The lifetime can be predicted from the obtained failure time data under the assumption that the physical law is empirically established between the stress and the lifetime and the logarithmic lifetime follows a specific probability distribution under a certain stress. In this research, focusing on finding the optimum sample allocation number, consider the case where the number of stress levels is three. By considering the linearity of the relational equation between stress and lifetime, we show a more reasonable interpretation of setting to 3 stress levels. In order to consider the reality, we use the mathematical model obtained from actual experimental data and its parameter value.

## Introduction

### Degradation and Lifetime

- In some of the electrically insulating materials, its physical properties are degraded due to external stress over time.
- For some materials such as the potting compound, lifetime is defined as occurring when physical properties is degraded below a certain threshold.
- We want to know the lifetime of the materials under the use stress using the Accelerated Degradation Test data.

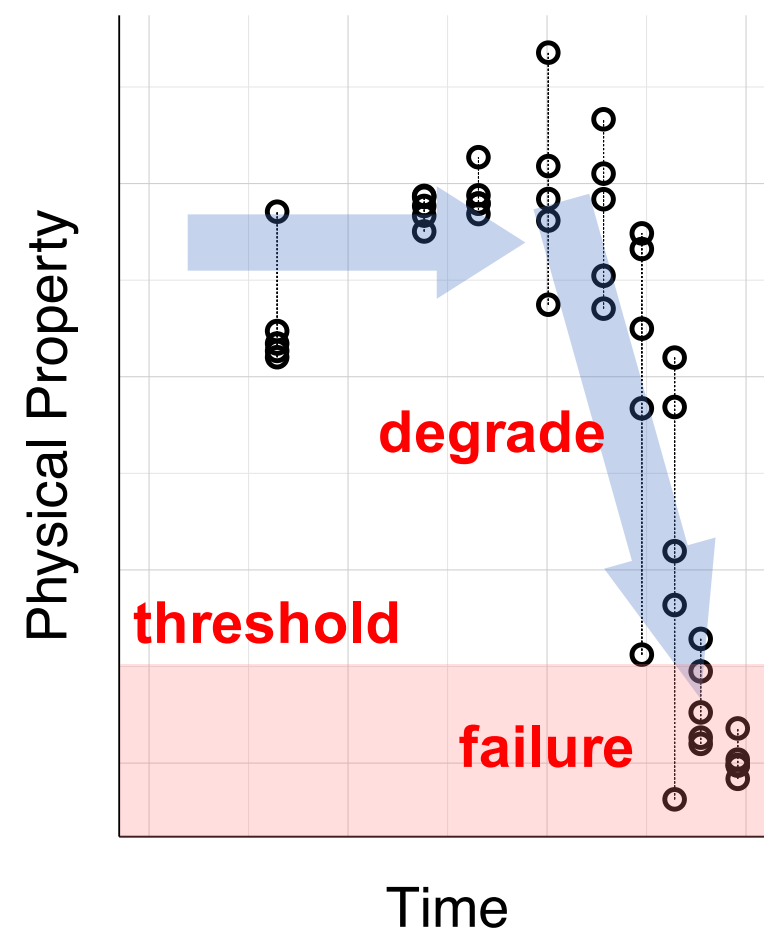


Figure 1: Relationship between Degradation and Lifetime

### Accelerated Degradation Test (ADT)

- ADT imposes a higher load on the target material than in actual use, and prompts degradation to obtain data in a short time.
- Observation data are physical properties of materials measured at a certain time.
- In order to know the accurate lifetime from the degradation information of fewer specimens, it is necessary to devise a test plan such as the number of stress levels, the number of specimens at each stress level, time interval to be measured and so on.
- Figure 2 shows an example for an ADT condition.

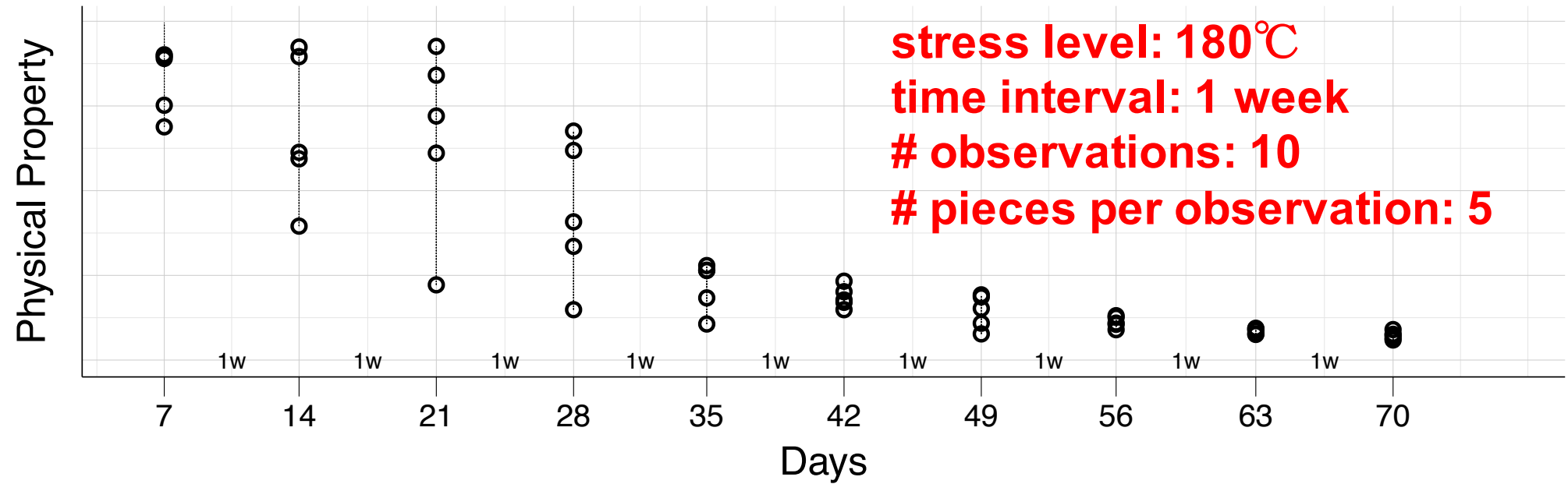


Figure 2: Example for an ADT data

### Related Works

Conventionally, some researches have been conducted on an optimum test plan for ADT. Shi, Escobar, and Meeker (2009) describes the optimal test plan for the degradation model according to the log-location scale family. Also, Liao and Tseng (2006) and Tseng, Balakrishnan, and Tsai (2009) describe the test plan of step stress ADT which gradually increases the stress level. Hirose and Sakumura (2015) proposed the Arrhenius Model for the Potting Compound data.

## Purpose

- In this research, we focus on the Arrhenius model of Hirose and Sakumura (ibid.), which is proposed as an accelerated degradation model for temperature stress.
- We express a method to verify the optimal test plan.
- Especially, we describe a compromise plan in case of considering three stress levels.

### ADT Data

In this study, we deal with Potting Compound data. Potting compound is a protective agent for insulation material. It protects against water, chemicals, shock, voltage discharge, vibration, and physical damage. Hirose and Sakumura (ibid.) provided data on the flexural strength against the thermal stress of the potting compound.



Figure 4: Ref.: Global Potting Compound Market 2017 — Dow Corning, Henkel AG & Co. KGAA, ACC Silicones Ltd., Elantas Beck Southeast Asia Ltd., Master Bond Inc.

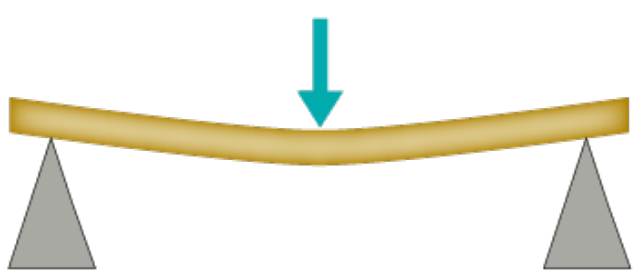


Figure 3: Ref.: Flexural Strength Epoxies — MasterBond.com

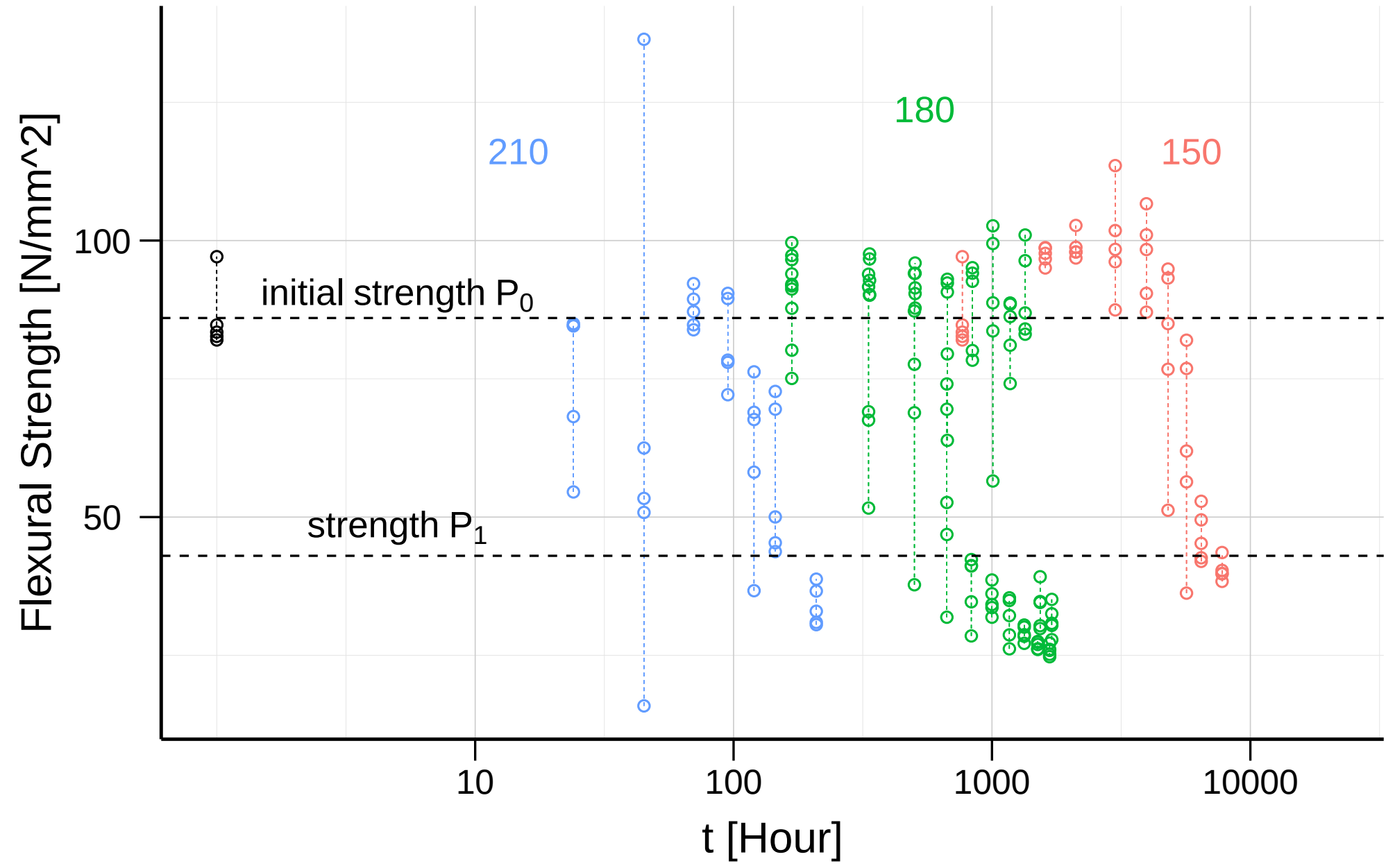


Figure 5: Degradation data of flexural strength for the potting compound due to thermal stress using ADT. Failure occurs when the physical properties fall below  $P_1$ .

This figure shows the Degradation of flexural strength due to thermal stress for Potting Compound Data.

- The number of stress levels is 3. The highest stress is 210 °C. The middle is 180 °C. The lowest is 150 °C. The horizontal axis is the logarithmic time.
- At 210 °C, there is data for about one week. At 150 °C, there is data for about one year. The actual temperature to use is lower, about 50 °C or less and less than 100 °C.
- In this test, the material imposes higher stress than actual and promotes degradation. We collect data in a short time and estimate the actual life from there.
- This data includes the non-stress data.
- From this non-stress data, we assume the initial strength  $P_0$ .
- And, then, Failure occurs when the physical properties fall below  $P_1$ .
- In this research, we deal with this data and consider the test plan under the thermal stress.

## Materials and Methods

### Degradation Model (ibid.)

- Let  $P$  denotes the amount of degradation and  $P_0$  denotes the initial physical properties.
- $P$  monotonically decreases from  $P_0$ .
- Let  $S(x)$  be the monotonically decreasing function and  $F(x) = 1 - S(x)$  as the cumulative distribution function at transformed time  $x$ . Thus,

$$P = P_0 S(x).$$

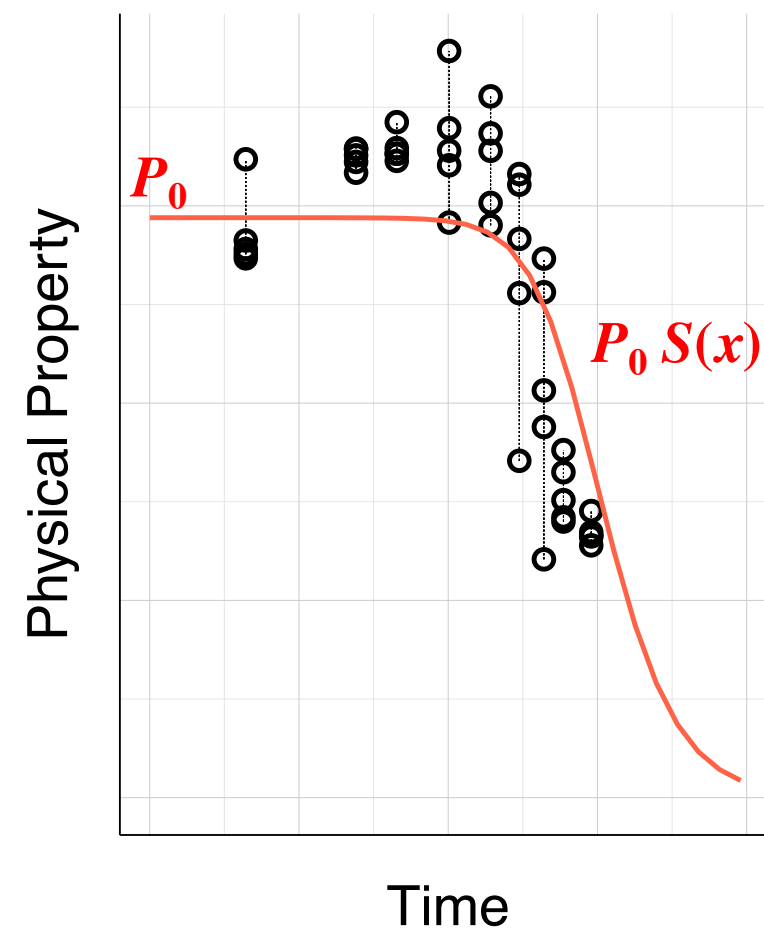


Figure 6: The concept of Degradation Model

### Arrhenius logistic Model (ibid.)

Suppose that  $x$  follows the generalized logistic distribution;

$$F(x) = \frac{1}{\{1 + \exp(-x)\}^\beta},$$

where

$$z = \frac{x - \mu}{\sigma}, \quad x = \log t, \quad \mu = v + \sigma \log \{p^{-1/\beta} - 1\}.$$

$v$  represents the Arrhenius law as follows;

$$v = S^{-1}(p) = \frac{E}{RT} - \log B,$$

The Arrhenius law is a empirical rule that holds between a lifetime and temperature.

- $p$  is the ratio to  $P_0$  which means the deterioration rate considered as a failure,
- $B$  is a constant,
- $E$  is a parameter called activation energy,
- $R$  is the Boltzmann constant  $8.6159 \times 10^{-5}$  [eV / K], and
- $T$  is the absolute temperature [K].

Here, Unknown paramter  $\theta = \{E, \log B, \sigma, \beta\}$ .

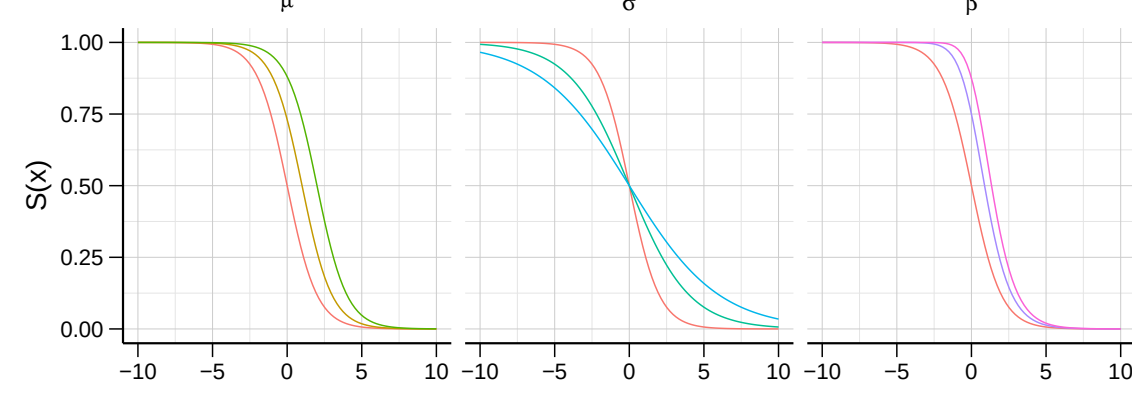


Figure 7: The generalized logistic distribution

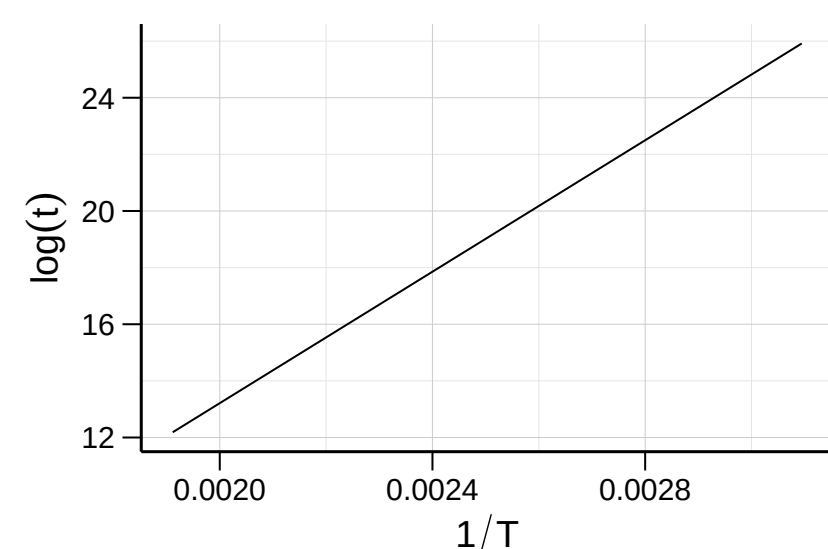


Figure 8: The Arrhenius law

### Likelihood

- The observed data is the stress level  $T_j$  ( $j = 1, \dots, m$ ) and the measured time  $t_{i,j}$  ( $i = 1, \dots, n_j$ ) where  $n_j$  is the number of samples for each level  $T_j$ .
- The indicator function  $r_{i,j}$  is  $r_{i,j} = 1$  for failure and  $r_{i,j} = 0$  for survival.
- Then the probability that the observed data will be generated is given by

$$L \propto \prod_{j=1}^m \prod_{i=1}^{n_j} \left[ F(x_i)^{r_{i,j}} \cdot \{1 - F(x_i)\}^{1-r_{i,j}} \right].$$

### Prediction and Asymptotic variance

- In this research, we consider as the optimal test plan when the asymptotic variance at the prediction value is minimized.
- The  $p_y$  percentile for failure at stress level  $T_y$  is

$$h = h(\theta_1, \theta_2, \dots, \theta_M) = \mu - \sigma \log \left( p_y^{-1/\beta} - 1 \right).$$

- The asymptotic variance at predicted lifetime  $h$  is obtained from error propagation rules (Ku 1966; Nelson 2009)  $\text{Var}(\hat{h}) = \mathbf{H} \mathbf{\Sigma} \mathbf{H}^T$  where  $\mathbf{\Sigma}$  represents the variance covariance matrix and

$$\mathbf{H} = \left( \frac{\partial h}{\partial \theta_1}, \frac{\partial h}{\partial \theta_2}, \dots, \frac{\partial h}{\partial \theta_M} \right).$$

- Here, when the stress levels and the model parameters are known, the optimum number of samples to be allocated to each stress level is obtained as minimizing  $\text{Var}(\hat{h})$ .

## Optimum Test Plan

### Asymptotic variance for Arrhenius Logistic model

- The variance covariance matrix can be calculate the inverse of Fisher information matrix  $\mathbf{\Sigma} = \mathbf{I}^{-1}$ .
- The fisher information matrix is given by the summation of the fisher information of each stress level.

$$\mathbf{I} = \sum_{i=1}^K I_i, \quad I_i = -E \left( \frac{\frac{\partial^2 L_i}{\partial E^2}}{\frac{\partial^2 L_i}{\partial \log B^2}}, \frac{\frac{\partial^2 L_i}{\partial E \partial \log B}}{\frac{\partial \log B \partial \sigma}{\partial^2}}, \frac{\frac{\partial^2 L_i}{\partial E \partial \sigma}}{\frac{\partial \log B \partial \beta}{\partial^2}}, \frac{\frac{\partial^2 L_i}{\partial E \partial \beta}}{\frac{\partial \sigma \partial \beta}{\partial^2}}, \frac{\frac{\partial^2 L_i}{\partial \log B \partial \sigma}}{\frac{\partial^2 L_i}{\partial \sigma^2}}, \frac{\frac{\partial^2 L_i}{\partial \log B \partial \beta}}{\frac{\partial^2 L_i}{\partial \beta^2}} \right)$$

### For example

- the MLE result for potting compound data is  $\{E = 1.18, \log B = 23.3, \sigma = 0.275, \beta = 1.61\}$ .
- Then, by using these parameters, we can consider the optimal test plan.
- For this result, when the actual stress level is 50°C and the percentile for failure is 0.5, then

$$\text{e.g.) } aVar = \mathbf{H} \mathbf{\Sigma} \mathbf{H}^T |_{T=50, p_y=0.5} = 0.252389$$

### Test plan for ADT of Three Stress Levels

- Here, we consider the test plan of ADT in this research.
- The components of ADT considered in this research are as follows:
  - The number of stress levels is 3,
  - Stress level is  $\{150^\circ\text{C}, 180^\circ\text{C}, 210^\circ\text{C}\}$ ,
  - Time interval is separated between lower and upper limit times at equal intervals such as  $\{\text{lower limit time, upper limit time}\} = \{[1440, 14400], [264, 2640], [24, 240]\}$  [Hour], and
  - The number of observations for each stress level  $\{n_L, n_M, n_H\}$  ( $n_L + n_M + n_H = 30$ ).
- We will investigate an allotment of specimens for each stress levels.

### Example for separating between minimum and maximum times at equal intervals

- $\{\text{lower limit time, upper limit time}\} = [1440, 14400]$  [Hours] in case of  $T = 150^\circ\text{C}$ .

If  $n_L = 10$ , the time interval becomes 1440 [Hours] = 2 [Month]

If  $n_L = 5$ , the time interval becomes 3240 [Hours] = 4.5 [Month]

| 1    | 2    | 3    | 4    | 5    | 6    | 7     | 8     | 9     | 10    |
|------|------|------|------|------|------|-------|-------|-------|-------|
| 1440 | 2880 | 4320 | 5760 | 7200 | 8640 | 10080 | 11520 | 12960 | 14400 |

| 1    | 2    | 3    | 4     | 5     |
|------|------|------|-------|-------|
| 1440 | 4680 | 7920 | 11160 | 14400 |

### Result of Optimal plan

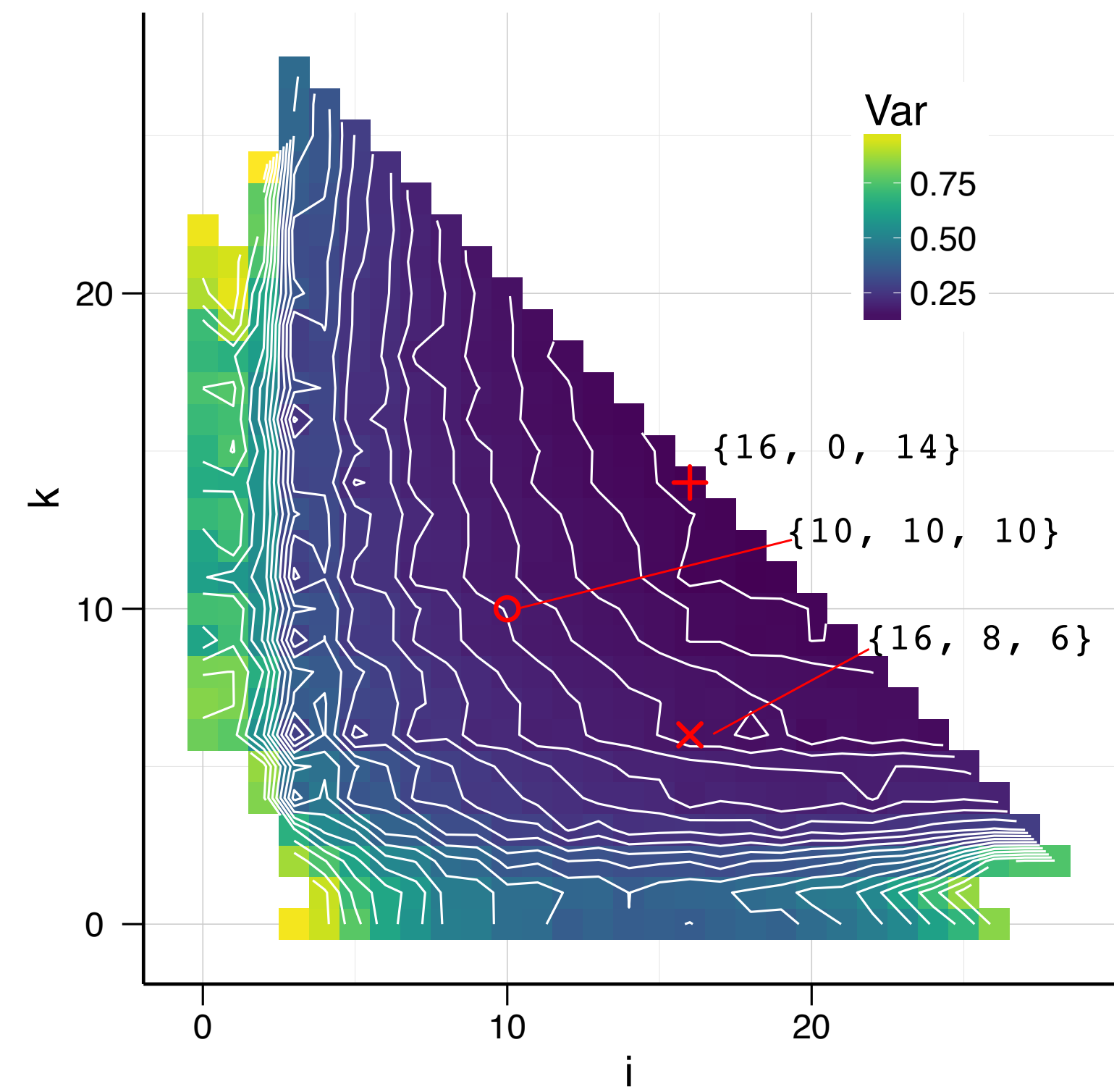


Figure 10: The result of optimum and compromised test plan

The result of optimum test plan is following;

- for lower stress  $T = 150^\circ\text{C}$ , the number of observation is 16 and its time interval is 36 [day] = 864 [hour],

| i | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11    | 12    | 13    | 14    | 15    | 16    |
|---|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|
| t | 1440 | 2304 | 3168 | 4032 | 4896 | 5760 | 6624 | 7488 | 8352 | 9216 | 10080 | 10944 | 11808 | 12672 | 13536 | 14400 |

- The middle stress,  $T = 180^\circ\text{C}$  is not allocated materials.
- for higher stress  $T = 210^\circ\text{C}$ , the number of observation is 14 and its time interval is 16.6 [hour].

| i | 1  | 2    | 3    | 4    | 5    | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14  |
|---|----|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| t | 24 | 40.6 | 57.2 | 73.8 | 90.5 | 107.1 | 123.7 | 140.3 | 156.9 | 173.5 | 190.2 | 206.8 | 223.4 | 240 |

- In other words, the optimum test plan is a test based on 2 stress levels.

### ADT of 3 stress levels

- However, in real situation, not 2 stress levels but 3 or more stress levels are used. Because it is necessary to investigate whether the Arrhenius law is established or not.
- Therefore, it is necessary to put constraint conditions so that it becomes 3 stress levels. (compromised plan)
- For example, it is assumed to have such order relation;  $n_L > n_M > n_H$ .
- Under these conditions, it became 16, 8, and 6 in order of low stress.
- For the result of compromised test plan,
- for  $T = 150^\circ\text{C}$ , time interval is 36 [day] = 864 [hour],

| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11    | 12    | 13    | 14    | 15    | 16    |
|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|
| 1440 | 2304 | 3168 | 4032 | 4896 | 5760 | 6624 | 7488 | 8352 | 9216 | 10080 | 10944 | 11808 | 12672 | 13536 | 14400 |

- for  $T = 180^\circ\text{C}$ , time interval is 2 [week] = 168 [hour],

| 1   | 2     | 3     | 4      | 5      | 6      | 7      | 8    |
|-----|-------|-------|--------|--------|--------|--------|------|
| 264 | 603.4 | 942.9 | 1282.3 | 1621.7 | 1961.1 | 2300.6 | 2640 |

- and for  $T = 210^\circ\text{C}$ , time interval is 43.2 [hour].

| 1  | 2    | 3     | 4     | 5     | 6   |
|----|------|-------|-------|-------|-----|
| 24 | 67.2 | 110.4 | 153.6 | 196.8 | 240 |

- An asymptotic relative efficiency based on the traditional test plan is  $aVar(optim)/aVar(eq) = 0.11/0.18 = 0.620$  for the optimum test plan and  $aVar(comp)/aVar(eq) = 0.15/0.18 = 0.838$  for the compromised test plan.

## Conclusion

- In this study, we consider the ADT test plan using the real data, called Potting Compound data, for thermal stress.
- We applied the Arrhenius Logistic model by Hirose and Sakumura (2015).
- The test plan is optimized by minimizing the asymptotic variance for the predicted value.
- The optimal test plan becomes the plan that two stress level consists of lowest and highest stress.
- In reality, since testing is conducted in three or more stress levels, it is possible to consider compromised test plan by adding constraints.

## References

Hirose, H. and T. Sakumura (2015). "Foundation of mathematical deterioration models for the thermal stress." *IEEE Transactions on Dielectrics and Electrical Insulation* 22.1, pp. 485–487.  
Ku, H. H. (1966). "Notes on the Use of Propagation of Error Formulas." *Journal of Research of the National Bureau of Standards* 70C.4, pp. 75–79.  
Liao, Chen-Mao and Sheng-Tsai Tseng (2006). "Optimal Design for Step-Stress Accelerated Degradation Tests." *IEEE Transactions on Reliability* 55.1, pp. 59–66.  
Nelson, Wayne B. (2009). "Accelerated testing: statistical models, test plans, and data analysis." 344. John Wiley & Sons.  
Shi, Ying, Luis A. Escobar, and William Q. Meeker (2009). "Accelerated Destructive Degradation Test Planning." *Technometrics* 51.1, pp. 1–13.  
Tseng, Sheng-Tsai, Narayanaswamy Balakrishnan, and Chih-Chun Tsai (2009). "Optimal Step-Stress Accelerated Degradation Test Plan for Gamma Degradation Processes." *IEEE Transactions on Reliability* 58.4, pp. 611–618.