

Parameter Estimation for the Modulated Extended Cumulative Exposure Model

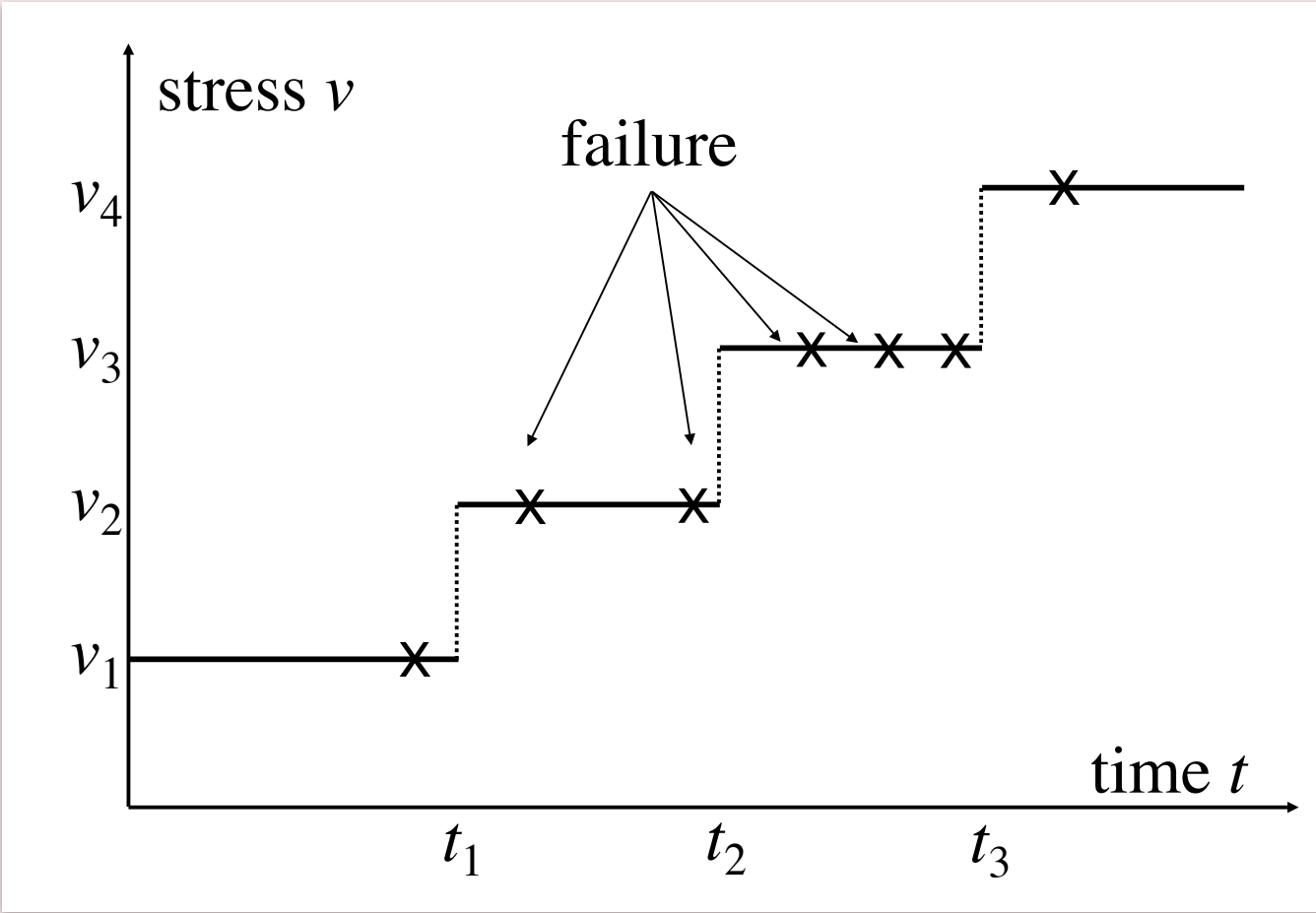
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Introduction: Step-stress accelerated life testing (SSALT) and its probability model



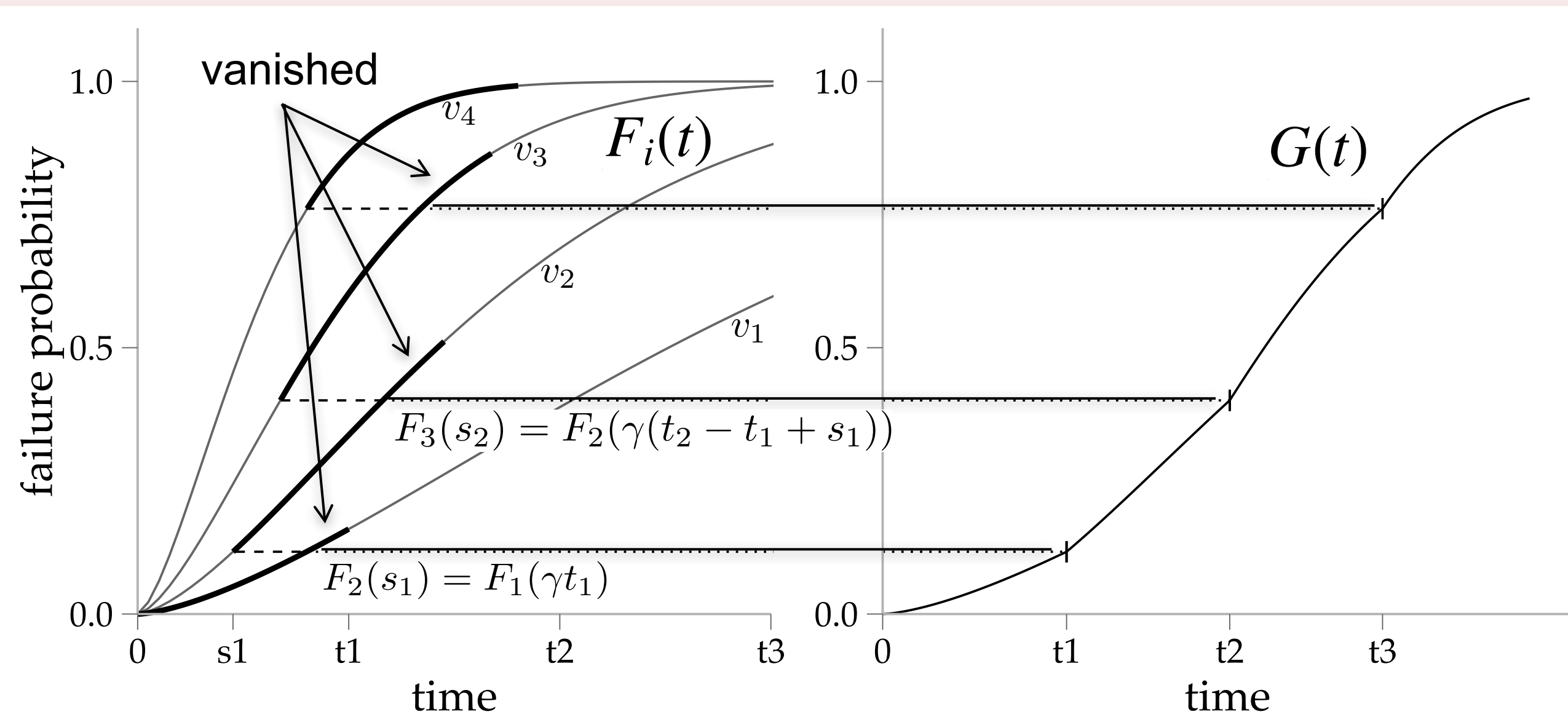
Focus: This study focuses on the **step-stress accelerated life testing (SSALT)** to find the lifetime of industrial materials (Nelson 1990) such as electrical insulation.

Objective: We have recently proposed the **modulated extended cumulative exposure model (modulated ECEM)** as a new SSALT model, however, this model is sometimes difficult to estimate its' parameters.

Implementation: This study demonstrates Markov Chain Monte Carlo (MCMC) techniques that are particularly well-suited for complex models such as the modulated ECEM.

Model: Modulated Extended Cumulative Exposure Model

We assume that the modulated ECEM vanishes the fatigue accumulation as **returning the imposed time** (Sakumura and Kamakura (2015)).



- The vanishing of the fatigue accumulation is defined as **the imposed time reversing**. A different step stress pattern has a different $G(t)$ distribution.

1:	$0 \leq t \leq t_1$	$G(t) = F_1(t)$	
2:	$t_1 \leq t \leq t_2$	$G(t) = F_2(\gamma(t - t_1 + s_1))$	$F_2(s_1) = F_1(\gamma t_1) \quad (0 \leq \gamma \leq 1)$
3:	$t_2 \leq t \leq t_3$	$G(t) = F_3(\gamma(t - t_2 + s_2))$	$F_3(s_2) = F_2(\gamma(t_2 - t_1 + s_1))$
:	:	:	:
i:	$t_{i-1} \leq t \leq t_i$	$G(t) = F_i(\gamma(t - t_{i-1} + s_{i-1}))$	$F_i(s_{i-1}) = F_{i-1}(\gamma(t_{i-1} - t_{i-2} + s_{i-2}))$

- If $F_i(t)$ is the Weibull power law model, then the CDF becomes

$$G(t) = F_i(t - t_{i-1} + s_{i-1}) = 1 - \exp[-\varepsilon(t)^\beta]$$

$$\varepsilon(t) = K^{-1} \left[(v_i - v_{th})^n (t - t_{i-1}) + \sum_{j=1}^{i-1} \gamma^j (v_{i-j} - v_{th})^n \Delta t_{i-j} \right].$$

- Here, N is the sample size, and $i(j)$ denotes that sample j is broken at level i . The likelihood function becomes to

$$L = \prod_{j=1}^N \left[G(t_{i(j)}) - G(t_{i(j)-1}) \right], \quad (1)$$

which provides the case for grouped data (we only observe the number of failures at each stress level).

The Weibull power law model

$$F(t; v_i) = F_i(t) = 1 - \exp \left\{ - (t / \{K(v_i - v_{th})^{-n}\})^\beta \right\}.$$

- t is the time to failure
- v_i is the applied stress at level i
- v_{th} : the threshold stress below which failures will not occur (Hirose (1993)).
- K : a constant parameter,
- n : the degradation rate,
- β is the shape parameter in the Weibull distribution
- $(k = K^{-1/n})$

Methodology: Parameter Estimation using Markov Chain Monte Carlo (MCMC) based on Metropolis-Hastings (M-H) algorithm within Gibbs sampling

- Let $X = (t, v)$ is the observed data, γ and $\theta = (K, n, \beta)$ is the model parameter, and $p(X|\gamma, \theta) = L$.

1. Attempt to draw $\gamma^k \sim p(\gamma|\theta^{k-1}, X)$

- Draw $\gamma^* \sim q_\gamma(\gamma^{k-1}, \gamma) \rightarrow$
- Accept $\gamma^k = \gamma^*$ with probability

$$\alpha(\gamma^{k-1}, \gamma^*) = \min \left\{ \frac{p(X|\gamma^*, \theta^{k-1})p(\gamma^*, \theta^{k-1})q_\gamma(\gamma^*, \gamma^{k-1})}{p(X|\gamma^{k-1}, \theta^{k-1})p(\gamma^{k-1}, \theta^{k-1})q_\gamma(\gamma^{k-1}, \gamma^*)}, 1 \right\},$$

Otherwise, set $\gamma^k = \gamma^{k-1}$.

2. Attempt to draw $\theta^k \sim p(\theta|\gamma^k, X)$

- Draw $\theta^* \sim q_\theta(\theta^{k-1}, \theta) \rightarrow$
- Accept $\theta^k = \theta^*$ with probability

$$\alpha(\theta^{k-1}, \theta^*) = \min \left\{ \frac{p(X|\theta^*, \gamma^{k-1})p(\theta^*, \gamma^{k-1})q_\theta(\theta^*, \theta^{k-1})}{p(X|\theta^{k-1}, \gamma^{k-1})p(\theta^{k-1}, \gamma^{k-1})q_\theta(\theta^{k-1}, \theta^*)}, 1 \right\},$$

Otherwise, set $\theta^k = \theta^{k-1}$.

$$q_\gamma(\gamma^{k-1}, \gamma) = \gamma^{k-1} + 0.2u, \\ u \sim \text{Unif}(-1, 1)$$

$$q_n(n^{k-1}, n^*) = n^{k-1} + u, \\ q_k(k^{k-1}, k^*) = k^{k-1} + 0.001u, \\ q_\beta(\beta^{k-1}, \beta^*) = \beta^{k-1} + 0.2u$$

$$p(\gamma) \sim \text{Beta}(0.5, 0.5), p(n) \sim \log\text{-normal}(2.2, 0.25), p(k) \sim N(0.05, 0.05), p(\beta) \sim \log\text{-normal}(0, 0.5)$$

Case Study - Step-Stress Test Data on Cable I Nelson (1980)

Specimen Data			
Hold Time (min)	Final Step	Total Time to Failure (min)	Thickness (mils)
15	9	102	27
15	9	113	27
15	9	113	27
60	10	370*	29.5
:	:	:	:

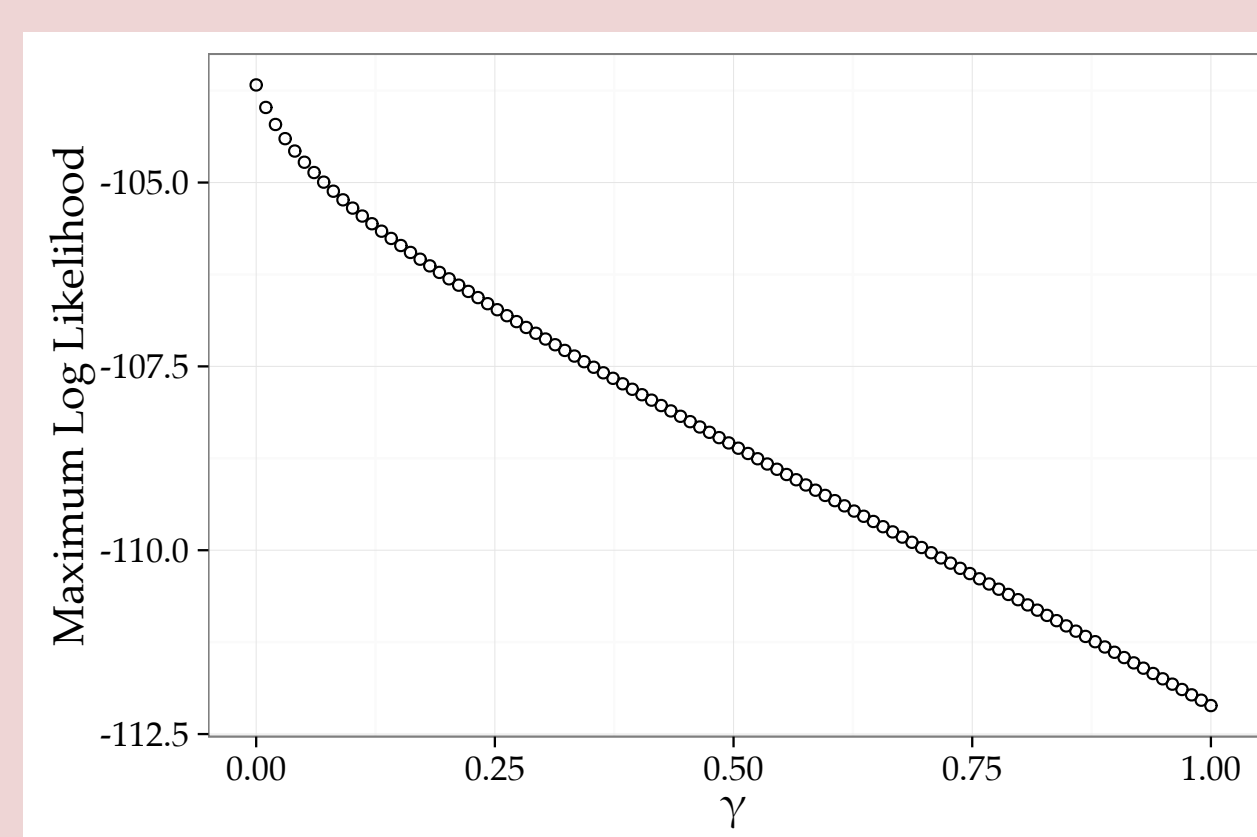


Figure: Maximum likelihood v.s. γ

Simulation Study

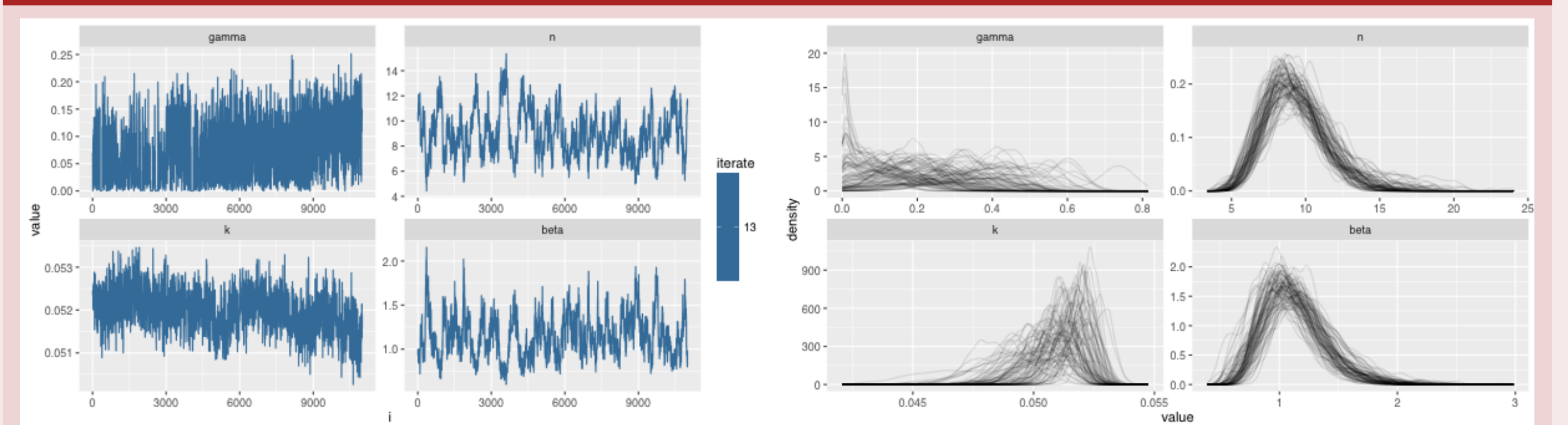


Figure: $n = 10, k = 0.0523, \beta = 1, \gamma = 0$

Table: Basic statistics for the estimated parameters in the simulation study.

n	k	β	γ		$\hat{\gamma}$	$\hat{n}$	$\hat{k}$	$\hat{\beta}$
10	0.0523	1	0	mean	0.219	9.37	0.0509	1.12
				sd	0.1096	0.4760	0.0008524	0.07073
			0.5	mean	0.522	9.51	0.0517	1.09
				sd	0.1159	0.4830	0.001140	0.06366
			1	mean	0.983	11.0	0.0547	0.879
				sd	0.01459	0.8409	0.001533	0.06265

Conclusion

Summary: In this paper, we have investigated the parameter estimation of the modulated ECEM using MCMC based on M-H within Gibbs. We have conducted a simulation study on a mimicked case and applied a real data.

Finding: We have successfully obtained suitable parameters.